ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 Complex Calculus

1st Midterm

March 29, 2010

17:40-19:30

Surname : _____

Name : _____

ID # : _____

Department : _____

Section : _____

Instructor : _____

Signature : _____

- The exam consists of 5 questions of equal weight.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

Question 1.

(a) Let $f(x+iy) = x^4 - 6x^2y^2 + y^4 + 4x^3yi - 4xy^3i + y^2i$. Find the region D

(i) where f is differentiable, (7 points)

(ii) where f is analytic. (3 points)

(b) Let $u(x,y) = x^2 - y^2 + x - y$.

(i) Show that u is harmonic everywhere. (4 points)

(ii) Find an analytic function f so that $\mathrm{Re}(f(z)) = u(x,y)$ and $f(1+i) = 0$? (6 points)

Answer 1.

Question 2.

(a) Evaluate $\lim_{n\to\infty} \dfrac{in^2 - in + 1 - 3i}{(2n + 4i - 3)(n - i)}$ if it exists. (10 points)

(b) Find the radius of convergence of the power series $\sum_{n=0}^{\infty}(1 + (-1)^n)^n z^n$. (10 points)

Answer 2.

Question 3.

(a) Solve the equation $e^{iz} - e^{-iz} = 2i$. (10 points)

(b) Find the image of the rectangle $\{(x, y) : \ 0 \leq x \leq 1, \ 0 \leq y \leq \frac{\pi}{4}\}$ under e^z. (10 points)

Answer 3.

Question 4.

(a) (i) Find all values of $\log(-\sqrt{3} - i)$. (5 points)

(ii) Find the principal value of $(1+i)^{2-i}$. (5 points)

(b) (i) Write

$$f(z) = \frac{z+i}{z^2+1}$$

in the form $u(x,y) + iv(x,y)$. (5 points)

(ii) Write $\left(\frac{1+i}{\sqrt{2}}\right)^{2010}$ in $a+bi$ form. (5 points)

Answer 4.

Question 5.

(a) Find the image of $|z-1|=1$ under the mapping $w=\frac{1}{z}$. (10 points)

(b) Let

$$f(z)=\begin{cases}\dfrac{z\,\mathrm{Re}(z)}{|z|} & z\neq 0,\\[2ex] 0 & z=0.\end{cases}$$

Show that f is continuous at the origin. (10 points)

Answer 5.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 Complex Calculus

2nd Midterm

May 3, 2010

17:40-19:30

Surname : _____

Name : _____

ID # : _____

Department : _____

Section : _____

Instructor : _____

Signature : _____

- The exam consists of 5 questions of equal weight.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
20	20	20	20	20	100

Question 1.

(a) Prove that

$$\arctan z = \frac{i}{2} \log \left(\frac{i+z}{i-z} \right).$$

(10 points)

(b) Solve the equation $\tan z = i\sqrt{3}$. (10 points)

Answer 1.

Question 2.

(a) Evaluate

$$\int_C e^{\bar{z}}\, dz,$$

where C is the line segment from the origin to the point $1 + i$. (10 points)

(b) Evaluate

$$\int_C \cos z\, dz,$$

where C is the polygonal path from 0 to $1 + i$ that consists of the line segments from 0 to 1 and 1 to $1 + i$. (10 points)

Answer 2.

Question 3.

(a) Let C be the circle $|z - 2i| = 1$. Evaluate

(i) $\displaystyle\oint_C \frac{z+i}{z^3+2z^2}\,dz.$ (5 points)

(ii) $\displaystyle\oint_C \frac{2}{2z-3i}\,dz.$ (5 points)

(b) Let C denote the boundary of the square whose sides along the lines $x = \pm 2$ and $y = \pm 2$. Evaluate

$$\oint_C \frac{\cos z}{z^2(z^2+8)}\,dz$$

(10 points)

Answer 3.

Question 4.

(a) Find the Laurent series expansion of $f(z) = \dfrac{1-z}{z-3}$ which is valid in $|z-1| > 2$.

(10 points)

(b) Find the first five terms of the Maclaurin series of $f(z) = \tan z$ which is valid in $|z| < \frac{\pi}{2}$.

(10 points)

Answer 4.

Question 5.

(a) Locate the zeros and poles of $f(z) = \dfrac{\sin z}{z^2 + z}$ and determine their orders. (10 points)

(b) Determine whether the function $f(z)$ defined by

$$f(z) = \begin{cases} z \sin\left(\frac{1}{z}\right) & \text{when} \quad z \neq 0 \\ \\ 0 & \text{when} \quad z = 0 \end{cases}$$

is continuous or discontinuous at $z = 0$. (10 points)

Answer 5.

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 Complex Calculus

Final

June 8, 2010

09:00-10:50

Surname : _____

Name : _____

ID # : _____

Department : _____

Section : _____

Instructor : _____

Signature : _____

- The exam consists of 5 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	TOTAL
30	20	20	30	20	120

Question 1. In each part, classify the isolated singularity of f at $z = 0$ and find the corresponding residue.

(a) $f(z) = \dfrac{1}{z + z^2}$. (10 points)

(b) $f(z) = z \cos\left(\dfrac{1}{z}\right)$. (10 points)

(b) $f(z) = \dfrac{z - \sin z}{z}$. (10 points)

Answer 1.

Question 2. In each part, evaluate

(a) $\displaystyle\oint_{|z|=3} \frac{1}{z^5 - z^3}\, dz.$ (10 points)

(b) $\displaystyle\oint_{|z|=1} \frac{z^6}{1 - 2z^8}\, dz.$ (10 points)

Answer 2.

Question 3. Evaluate (20 points)

$$\int_0^{2\pi} \frac{\sin\theta}{2-\cos\theta}\, d\theta.$$

Answer 3.

Question 4. In each part, evaluate

(a) P.V. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^4-1}\,dx.$ (15 points)

(b) P.V. $\displaystyle\int_{0}^{\infty} \frac{\cos x}{x^4-1}\,dx.$ (15 points)

Answer 4.

Question 5.

(a) Determine the number of roots of the equation $z^6 - 5z^4 + 10 = 0$ on

(i) $|z| < 1$. (3 points)

(ii) $|z| < 2$. (3 points)

(iii) $|z| < 3$. (3 points)

(iv) $|z| > 3$. (3 points)

(b) Determine the value of $\triangle_C \arg f(z)$ if C is the circle $|z| = 2$, described in the positive sense and $f(z) = \dfrac{(z^3+2)(z-1)}{z^5(z^2+5)}$. (8 points)

Answer 5.