

ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 - Complex Calculus

FIRST MIDTERM EXAMINATION

26.03.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 5 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) Sketch the set of points determined by

(i) $|z - 1|^2 + |z + 1|^2 < 8$, (5 points)

(ii) $0 < \text{Arg}(z - 1 - i) < \frac{\pi}{3}$. (5 points)

(b) Find all solutions of the equation $z^4 = -8 + i8\sqrt{3}$ in rectangular coordinates and mark them on the complex plane. (10 points)

Answer 1.

Question 2.

(a) Let $f(z) = 4(\operatorname{Re} z)(\operatorname{Im} z) - i(\bar{z})^2$.

(i) Where is f differentiable? (5 points)

(ii) Where is f analytic? (3 points)

(iii) Calculate f' at the points of differentiability. (2 points)

(b) Let $u(x, y) = xy + 3x^2y - y^3$.

(i) Show that u is harmonic everywhere. (3 points)

(ii) Find a harmonic conjugate v of u . (5 points)

(iii) Express $f = u + iv$ as a function of z . (2 points)

Answer 2.

Question 3.

(a) Find $\lim_{n \rightarrow \infty} \left(\frac{1}{2} + \frac{i}{4} \right)^n$. (10 points)

(b) Find the radius and the disk of convergence of the power series $\sum_{n=0}^{\infty} \frac{(z-i)^n}{(3+4i)^n}$. (10 points)

Answer 3.

Question 4.

(a) Solve the equation $e^z = -ie$. (10 points)

(b) (i) Find all values of $(-1 + i\sqrt{3})^{\frac{3}{2}}$. (6 points)

(ii) Find the principal value of $(-1 + i\sqrt{3})^{\frac{3}{2}}$. (4 points)

Answer 4.

Question 5.

(a) Describe the image of

(i) the circle $|z| = 1$,

(8 points)

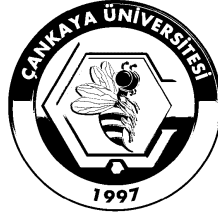
(ii) the disk $|z| < 1$

(4 points)

under the transformation $w = \frac{iz - i}{z + 1}$.

(b) Let \mathcal{D} be the part of the annulus $e^2 < |z| < e^3$ in the open third quadrant ($\operatorname{Re} z < 0$, $\operatorname{Im} z < 0$). Find the image of the \mathcal{D} under the principal logarithm function. (8 points)

Answer 5.



ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 - Complex Calculus

SECOND MIDTERM EXAMINATION

30.04.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 5 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) Prove that

$$\arccos z = -i \log \left(z + i(1 - z^2)^{\frac{1}{2}} \right).$$

(10 points)

(b) Solve the equation $\cos z = \frac{3}{4}i$.

(10 points)

Answer 1.

Question 2.

(a) Evaluate the integral $\int_C \bar{z} dz$ where C is the part of the curve $y = x^3$ from $z_1 = -2 - 8i$ to $z_2 = 1 + i$. (10 points)

(b) Evaluate

$$\int_C \text{Log } z dz$$

where C is the line segment from 1 to $1 + i$. (10 points)

Answer 2.

Question 3.

(a) Let C be the circle $|z - i| = 1$. Evaluate

(i) $\oint_C \frac{2z}{z^2 + 1} dz,$ (5 points)

(ii) $\oint_C \sec z dz.$ (5 points)

(b) Let C be the circle $|z| = 1$. Evaluate

$$\oint_C \frac{z + 3}{z^3 + 2z^2} dz.$$

(10 points)

Answer 3.

Question 4.

(a) Find the Laurent series expansion of $f(z) = \frac{2}{-z^2 + 4z - 3}$ which is valid in $1 < |z| < 3$. (10 points)

(b) Find the Maclaurin series of $f(z) = \cosh z$ and state where it is valid. (10 points)

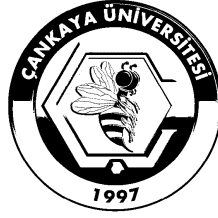
Answer 4.

Question 5.

(a) Locate the singularities of $f(z) = ze^{\frac{1}{z}}$ and determine their type. (10 points)

(b) Locate the zeros and poles of $f(z) = \frac{z^2}{e^z - 1}$ and determine their order. (10 points)

Answer 5.



ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 - Complex Calculus

FINAL EXAMINATION

28.05.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 120 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Bonus		20
Total		120

IMPORTANT NOTES:

- 1) Please make sure that you have written your student number and name above.
- 2) Check that the exam paper contains 6 problems.
- 3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. In each part, find and classify all the isolated singularities of f and find the corresponding residues.

(a) $f(z) = \frac{1}{z^7 - z^9}$. (8 points)

(b) $f(z) = z^3 \sin \frac{1}{z^2}$. (6 points)

(c) $f(z) = \frac{e^z - 1 - z}{z^2}$. (6 points)

Answer 1.

Question 2. In each part, evaluate

(a) $\oint_{|z|=2} \frac{e^z}{z^2 - 4z + 3} dz.$ (10 points)

(b) $\oint_{|z|=2} \frac{1}{(z^2 - 1)(z^2 - 2)(z^2 - 3)} dz.$ (10 points)

Answer 2.

Question 3. Evaluate

(20 points)

$$\int_0^{2\pi} \frac{\cos \theta}{5 + 3 \cos \theta} d\theta.$$

Answer 3.

Question 4. In each part, use residues to evaluate

(a) $\int_0^{\infty} \frac{1}{(x^2 + 1)^2} dx.$ (10 points)

(b) $\int_{-\infty}^{\infty} \frac{x \sin \pi x}{x^2 + 2x + 5} dx.$ (10 points)

Answer 4.

Question 5.

(a) Let $f(z) = \frac{(z^2 + 1)^2}{(z^2 + 2z + 2)^3}$. Evaluate $\frac{1}{2\pi i} \oint_{|z|=4} \frac{f'(z)}{f(z)} dz$. (10 points)

(b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$. (10 points)

Answer 5.

Bonus. Use residues to evaluate $\int_0^\infty \frac{dx}{x^{\frac{1}{3}}(1+x)}$. (20 points)

Answer.