ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MCS 352 - Complex Calculus

FIRST MIDTERM EXAMINATION
26.03.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) Sketch the set of points determined by

(i) $|z-1|^2+|z+1|^2<8$, (5 points)

(ii) $0<\mathrm{Arg}(z-1-i)<\dfrac{\pi}{3}$. (5 points)

(b) Find all solutions of the equation $z^4=-8+i8\sqrt{3}$ in rectangular coordinates and mark them on the complex plane. (10 points)

Answer 1.

Question 2.

(a) Let $f(z) = 4(\operatorname{Re} z)(\operatorname{Im} z) - i(\bar{z})^2$.

(i) Where is f differentiable? (5 points)

(ii) Where is f analytic? (3 points)

(iii) Calculate f' at the points of differentiability. (2 points)

(b) Let $u(x, y) = xy + 3x^2y - y^3$.

(i) Show that u is harmonic everywhere. (3 points)

(ii) Find a harmonic conjugate v of u. (5 points)

(iii) Express $f = u + iv$ as a function of z. (2 points)

Answer 2.

Question 3.

(a) Find $\lim_{n\to\infty}\left(\frac{1}{2}+\frac{i}{4}\right)^n$. (10 points)

(b) Find the radius and the disk of convergence of the power series $\sum_{n=0}^{\infty}\frac{(z-i)^n}{(3+4i)^n}$. (10 points)

Answer 3.

Question 4.

(a) Solve the equation $e^z = -ie$. (10 points)

(b) (i) Find all values of $(-1 + i\sqrt{3})^{\frac{3}{2}}$. (6 points)

(ii) Find the principal value of $(-1 + i\sqrt{3})^{\frac{3}{2}}$. (4 points)

Answer 4.

Question 5.

(a) Describe the image of

(i) the circle $|z| = 1$, (8 points)

(ii) the disk $|z| < 1$ (4 points)

under the transformation $w = \dfrac{iz - i}{z + 1}$.

(b) Let $\mathscr{D}$ be the part of the annulus $e^2 < |z| < e^3$ in the open third quadrant ($\operatorname{Re} z < 0$, $\operatorname{Im} z < 0$). Find the image of the $\mathscr{D}$ under the principal logarithm function. (8 points)

Answer 5.

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Department of Mathematics and Computer Science

MCS 352 - Complex Calculus

SECOND MIDTERM EXAMINATION
30.04.2012

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 110 minutes

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Total		100

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 5 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1.

(a) Prove that

$$\arccos z = -i \log \left(z + i(1 - z^2)^{\frac{1}{2}} \right).$$

(10 points)

(b) Solve the equation $\cos z = \dfrac{3}{4} i$. (10 points)

Answer 1.

Question 2.

(a) Evaluate the integral $\int_C \bar{z}\, dz$ where C is the part of the curve $y = x^3$ from $z_1 = -2 - 8i$ to $z_2 = 1 + i$. (10 points)

(b) Evaluate

$$\int_C \operatorname{Log} z\, dz$$

where C is the line segment from 1 to $1 + i$. (10 points)

Answer 2.

Question 3.

(a) Let C be the circle $|z - i| = 1$. Evaluate

(i) $\displaystyle\oint_C \frac{2z}{z^2+1}\,dz,$ (5 points)

(ii) $\displaystyle\oint_C \sec z\,dz.$ (5 points)

(b) Let C be the circle $|z| = 1$. Evaluate

$$\oint_C \frac{z+3}{z^3+2z^2}\,dz.$$

(10 points)

Answer 3.

Question 4.

(a) Find the Laurent series expansion of $f(z) = \dfrac{2}{-z^2 + 4z - 3}$ which is valid in $1 < |z| < 3$. (10 points)

(b) Find the Maclaurin series of $f(z) = \cosh z$ and state where it is valid. (10 points)

Answer 4.

Question 5.

(a) Locate the singularities of $f(z) = ze^{\frac{1}{z}}$ and determine their type. (10 points)

(b) Locate the zeros and poles of $f(z) = \dfrac{z^2}{e^z - 1}$ and determine their order. (10 points)

Answer 5.

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MCS 352 - Complex Calculus

FINAL EXAMINATION
28.05.2012

Question	Grade	Out of
1		20
2		20
3		20
4		20
5		20
Bonus		20
Total		120

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR: Seçil Gergün

DURATION: 120 minutes

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. In each part, find and classify all the isolated singularities of f and find the corresponding residues.

(a) $f(z) = \dfrac{1}{z^7 - z^9}$. (8 points)

(b) $f(z) = z^3 \sin \dfrac{1}{z^2}$. (6 points)

(c) $f(z) = \dfrac{e^z - 1 - z}{z^2}$. (6 points)

Answer 1.

Question 2. In each part, evaluate

(a) $\displaystyle\oint_{|z|=2} \frac{e^z}{z^2-4z+3}\,dz.$ (10 points)

(b) $\displaystyle\oint_{|z|=2} \frac{1}{(z^2-1)(z^2-2)(z^2-3)}\,dz.$ (10 points)

Answer 2.

Question 3. Evaluate (20 points)

$$\int_0^{2\pi} \frac{\cos\theta}{5+3\cos\theta}\, d\theta.$$

Answer 3.

Question 4. In each part, use residues to evaluate

(a) $\displaystyle\int_0^\infty \frac{1}{(x^2+1)^2}\,dx.$ (10 points)

(b) $\displaystyle\int_{-\infty}^\infty \frac{x\sin \pi x}{x^2+2x+5}\,dx.$ (10 points)

Answer 4.

Question 5.

(a) Let $f(z) = \dfrac{(z^2+1)^2}{(z^2+2z+2)^3}$. Evaluate $\dfrac{1}{2\pi i} \displaystyle\oint_{|z|=4} \dfrac{f'(z)}{f(z)}\, dz$. (10 points)

(b) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$. (10 points)

Answer 5.

Bonus. Use residues to evaluate $\displaystyle\int_0^\infty \frac{dx}{x^{\frac{1}{3}}(1+x)}$. (20 points)

Answer.